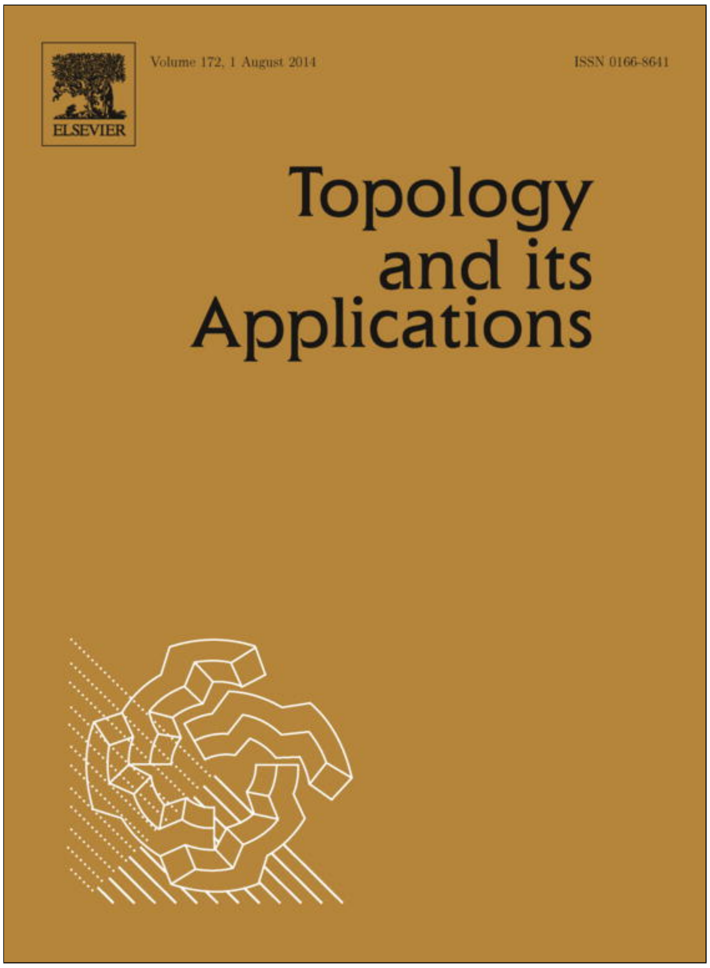
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Some uniformities on X related to topological properties of $C(X)$

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ABSTRACT

If X is a Hausdorff completely regular space, we characterize those spaces $C_c(X)$ whose compact sets are metrizable in terms of a particular uniformity on X. This fact is used to show that for a k-space X are equivalent (i) X satisfies the discrete countable chain condition, (ii) every admissible uniformity on X is trans-separable, and (iii) every compact set of $C_c(X)$ is metrizable. Several examples examine this result.

1. Preliminaries

In what follows, unless otherwise stated, X will be a Hausdorff completely regular space and $C_p(X)$ and $C_c(X)$ will denote the space $C(X)$ of all real-valued continuous functions defined on X provided with the pointwise convergence topology τ_p and with the compact-open topology τ_c, respectively. The topological dual of $C_p(X)$ will be denoted by $L(X)$, or by $L_p(X)$ when equipped with the weak* topology. For a complete account of the different uniform convergence topologies on $C(X)$, see [17].

A family $\{A_\alpha : \alpha \in \mathbb{N}^\mathbb{N}\}$ of subsets of a set X is called a *resolution* of X if $\bigcup\{A_\alpha : \alpha \in \mathbb{N}^\mathbb{N}\} = X$ and $A_\alpha \subseteq A_\beta$ whenever $\alpha \leq \beta$, [13, Chapter 3]. A locally convex space E belongs to the *class* $\mathfrak{G}$ if its topological dual E' has a resolution $\{A_\alpha : \alpha \in \mathbb{N}^\mathbb{N}\}$ such that for every $\alpha \in \mathbb{N}^\mathbb{N}$ each sequence in A_α is equicontinuous [13, Chapter 11]. Each compact set of a space $C_c(X)$ that belongs to the class $\mathfrak{G}$ is metrizable [13, Theorem 11.1]. If X is a separable metric space which is not Baire the space $C_c(X)$ does not belong to $\mathfrak{G}$ but, since it is submetrizable, every compact set in $C_c(X)$ is metrizable. Recall that a family $\mathcal{F}$ of

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functions from a uniform space $(X, \mathcal{N})$ into a uniform space $(Y, \mathcal{M})$ is called *uniformly equicontinuous* [3, X.2.1 Definition 2] if for each $V \in \mathcal{M}$ there is $U \in \mathcal{N}$ such that $(f(x), f(y)) \in V$ whenever $f \in \mathcal{F}$ and $(x, y) \in U$.

A uniform space $(X, \mathcal{N})$ is called *trans-separable* if for each vicinity N of $\mathcal{N}$ there is a countable subset Q of X such that $N[Q] = \bigcup_{x \in Q} U_N(x) = X$, where $U_N(x) = \{y \in X : (x, y) \in N\}$, see [13, Section 6.4]. Equivalently, $(X, \mathcal{N})$ is trans-separable if each uniform cover of X admits a countable subcover [12]. The word trans-separable was coined by Lech Drewnowski in [6]. Separable uniform spaces and Lindelöf uniform spaces are trans-separable and for uniform pseudometrizable spaces trans-separability is equivalent to separability. Equivalently, a (Hausdorff) uniform space $(X, \mathcal{N})$ is trans-separable if it is uniformly isomorphic to a subspace of a uniform product of separable pseudometric (metric) spaces. Each locally convex space in its weak topology is trans-separable when equipped with the (unique) admissible translation-invariant uniformity associated to its locally convex structure. The class of trans-separable uniform spaces is hereditary, productive and closed under uniform continuous images. Some useful applications of trans-separable spaces can be found in [13, Section 6.4].

A topological space satisfies the *Discrete Countable Chain Condition* (DCCC for short) [22] if every discrete family of open sets is countable. Separable spaces, countably compact spaces and Lindelöf spaces satisfy the DCCC. Further, each paracompact space that satisfies the DCCC is Lindelöf. Hence every metrizable space with the DCCC is separable. Continuous images of DCCC spaces satisfy the DCCC, but the DCCC property is not hereditary and the product of a family $\mathcal{F}$ of spaces that satisfy the DCCC satisfies the DCCC if every product of finitely many members of $\mathcal{F}$ does. If $(X, \mathcal{N})$ is a uniform space such that $(X, \tau_{\mathcal{N}})$ satisfies the DCCC, where $\tau_{\mathcal{N}}$ is the uniform topology, then $(X, \mathcal{N})$ is trans-separable [10]. If a completely regular DCCC space X has a subspace Y which does not satisfy the DCCC and $\mathcal{N}$ is an admissible uniformity on X, then Y with the relative uniformity $\mathcal{M}$ inherited from $\mathcal{N}$ is trans-separable but $(Y, \tau_{\mathcal{M}})$ fails to satisfy the DCCC.

The research on the space $C_c(X)$, which started with Warner's paper [20], is still active as evidenced by references [4,7,10,14]. We quote the following results for later use.

Theorem 1. *([10, Theorem 1]) A uniform space $(X, \mathcal{N})$ is trans-separable if and only if every pointwise bounded uniformly equicontinuous set of functions from $(X, \mathcal{N})$ into $\mathbb{R}$ is metrizable in $C_c(X, \tau_{\mathcal{N}})$.*

Theorem 2. *([5, Theorem 4]) A topological space satisfies the DCCC if and only if every pointwise bounded equicontinuous subset of $C(X)$ is τ_p-metrizable.*

With the help of Ascoli's theorem, easily follows from Theorem 2 that for a k-space X every compact set of $C_c(X)$ is metrizable if and only if X satisfies the DCCC. In what follows we extend this property by characterizing those spaces $C_c(X)$ whose compact sets are metrizable in terms of a particular uniformity $\mathcal{M}$ on X and use this to prove that if X is a k-space every compact set of $C_c(X)$ is metrizable if and only if X satisfies the DCCC if and only if every admissible uniformity on X is trans-separable. The equivalence of the latter two statements highlights the role of trans-separability to characterize certain topological properties in terms of admissible uniformities. Recall for instance that a completely regular space X is realcompact if and only if there exists an admissible uniformity $\mathcal{N}$ on X such that $(X, \mathcal{N})$ is trans-separable and complete [13, Proposition 6.9].

2. Characterization of the metrizability of compact sets

We start by characterizing those $C_c(X)$ spaces whose compact sets are metrizable in terms of a particular uniformity on X.

Theorem 3. *The compact sets of $C_c(X)$ are metrizable if and only if $(X, \mathcal{M})$, where $\mathcal{M}$ is the uniformity on X generated by the pseudometrics*

$$d_A(x, y) = \sup_{f \in A} \bigl| f(x) - f(y) \bigr| \tag{2.1}$$

for each compact set A of $C_c(X)$, is trans-separable.

Proof. Let E be the topological dual of $C_c(X)$. Let us denote by $\mathcal{K}(C_c(X))$ the family of all compact sets of $C_c(X)$ and by $\rho(E, C_c(X))$ the locally convex topology on E of uniform convergence on all compact sets of $C_c(X)$. If δ stands for the canonical homeomorphic embedding of X into $L_p(X)$, note that $\delta(X) \subseteq L(X) \subseteq E$ and observe that the topology $\rho_X = \delta^{-1}(\rho(E, C_c(X)))$ on X is stronger than the original topology of X, since $\rho(E, C_c(X))|_{\delta(X)}$ is stronger than the topology induced on $\delta(X)$ by the weak* topology $\sigma(E, C(X))$ of E. This latter fact implies that $C(X) \subseteq C(X, \rho_X)$ algebraically.

Assuming that all compact sets of $C_c(X)$ are metrizable, then the topological dual E of $C_c(X)$ equipped with the locally convex topology $\rho(E, C_c(X))$ is trans-separable, [9, Theorem 2]. This topology $\rho(E, C_c(X))$ generates a unique admissible translation-invariant uniformity $\mathcal{N}$ on E such that $\tau_{\mathcal{N}} = \rho(E, C_c(X))$. Setting $\widetilde{f}$ instead of $f \in C(X)$ when f is considered as a linear functional on E, observe that $(u, v) \in N \in \mathcal{N}$ with $u, v \in E$ if and only if there are $A \in \mathcal{K}(C_c(X))$ and $\epsilon > 0$ such that

$$\sup_{f \in A} \bigl| \langle \widetilde{f}, u - v \rangle \bigr| < \epsilon.$$

Particularly, the relative uniformity $\mathcal{M}_\delta$ of $\mathcal{N}$ on $\delta(X)$ satisfies that

$$(\delta_x, \delta_y) \in N \cap \bigl(\delta(X) \times \delta(X)\bigr) \quad \Leftrightarrow \quad \sup_{f \in A} \bigl| \langle \widetilde{f}, \delta_x \rangle - \langle \widetilde{f}, \delta_y \rangle \bigr| < \epsilon.$$

This defines a uniformity $\mathcal{M}$ on X such that $(x, y) \in M \in \mathcal{M}$ if and only if there are $A \in \mathcal{K}(C_c(X))$ and $\epsilon > 0$ with $\sup_{f \in A} |f(x) - f(y)| < \epsilon$. Given that $(X, \mathcal{M})$ and $(\delta(X), \mathcal{M}_\delta)$ are clearly uniformly isomorphic and, as mentioned before, the class of trans-separable spaces is hereditary and closed under uniform continuous images, it follows that the uniform space $(X, \mathcal{M})$ is trans-separable.

Assume conversely that $(X, \mathcal{M})$ is trans-separable when $\mathcal{M}$ is the uniformity on X generated by the pseudometrics $d_A(x, y) = \sup_{f \in A} |f(x) - f(y)|$ for every $A \in \mathcal{K}(C_c(X))$ and let A be a fixed compact subset of $C_c(X)$. As observed above, A is contained in $C(X, \tau_{\mathcal{M}}) = C(X, \rho_X)$. Moreover, A is a (pointwise bounded) uniformly equicontinuous set of functions from $(X, \mathcal{M})$ to $\mathbb{R}$ since, given $\epsilon > 0$, if $d_A(x, y) < \epsilon$ obviously $|f(x) - f(y)| < \epsilon$ whenever $f \in A$. Consequently, by Theorem 1, the set A is metrizable in $C_c(X, \tau_{\mathcal{M}})$. Since $\tau_{\mathcal{M}} = \rho_X$ is stronger than the original topology on X, there are as many compact sets in X as in $(X, \tau_{\mathcal{M}})$ or more, which ensures that the topology on A inherited from $C_c(X)$ is stronger than the corresponding one inherited from $C_c(X, \tau_{\mathcal{M}})$. Since A is compact in $C_c(X)$, both topologies coincide and A is metrizable. □

Theorem 4. *If X is a completely regular k-space, the following are equivalent*

(1) *X satisfies the DCCC.*
(2) *Every admissible uniformity on X is trans-separable.*
(3) *All compact sets of $C_c(X)$ are metrizable.*

Proof. (1) $\Rightarrow$ (2). Let $\mathcal{N}$ be an admissible uniformity on X, so that the uniform topology $\tau_{\mathcal{N}}$ coincides with the original topology of X. Let $\mathcal{D}$ be the family of all uniformly continuous pseudometrics on X. If $d \in \mathcal{D}$ and $V_{d,\epsilon} := \{(x, y) \in X \times X : d(x, y) < \epsilon\}$, then $V_{d,\epsilon} \in \mathcal{N}$ so that the identity map $id : (X, \tau_{\mathcal{N}}) \to (X, d)$

is continuous. Hence if $(X, \tau_{\mathcal{N}})$ satisfies the DCCC, the pseudometric space (X, d) is separable. If Q_d is a countable dense subset of (X, d), clearly $V_{d,\epsilon}[Q_d] = X$. Since for each $N \in \mathcal{N}$ there is $d \in \mathcal{D}$ such that $V_{d,1/4} \subseteq N$ (see [15, Chapter 6]), it follows that $N[Q_d] = X$. Hence $(X, \mathcal{N})$ is trans-separable.

(2) $\Rightarrow$ (3). Let us assume that every admissible uniformity on X is trans-separable. Let $\mathcal{M}$ be the uniformity on X generated by the pseudometrics (2.1) when A runs over the compact sets of $C_c(X)$. We claim that X and $(X, \tau_{\mathcal{M}})$ have the same compact sets. First note that, if δ and E denote respectively the canonical embedding of X into $L_p(X)$ and the topological dual of $C_c(X)$, then $(X, \tau_{\mathcal{M}})$ is homeomorphic to $\delta(X)$ when equipped with the relative topology of $\rho(E, C_c(X))$ on E of uniform convergence on all compact sets of $C_c(X)$. If K is a compact set of X, then the set

$$K^{\diamond} := \left\{ f \in C(X) : \sup_{x \in K} \left| f(x) \right| \leq 1 \right\}$$

is a neighborhood of the origin in $C_c(X)$ and, consequently, the absolutely convex set

$$K^{\diamond 0} := \left\{ u \in E : \sup_{f \in K^{\diamond}} \left| \langle \widetilde{f}, u \rangle \right| \leq 1 \right\}$$

is a weak* compact subset of E, that is, a $\sigma(E, C(X))$-compact set. Since $\rho(E, C_c(X))$ coincides with $\sigma(E, C(X))$ on the $C_c(X)$-equicontinuous subsets of E, it follows that $K^{\diamond 0}$ is a $\rho(E, C_c(X))$-compact subset of E. Now, since clearly $\delta(K)$ is a weak* closed subspace of $K^{\diamond 0}$, we have that $\delta(K)$ is $\rho(E, C_c(X))$-compact. Thus K is a $\tau_{\mathcal{M}}$-compact space. On the other hand, if K is a $\tau_{\mathcal{M}}$-compact set of X, since $\tau_{\mathcal{M}}$ is stronger than the original topology of X, the set K is compact in X.

If X is a k-space, the claim we have just proved ensures that the uniform topology $\tau_{\mathcal{M}}$ coincides with the original topology of X, so that $\mathcal{M}$ is an admissible uniformity on X. Since according to our assumptions $(X, \mathcal{M})$ is trans-separable, an application of Theorem 3 guarantees that every compact set of $C_c(X)$ must be metrizable.

(3) $\Rightarrow$ (1). Assume that every compact set of $C_c(X)$ is metrizable. Since every pointwise bounded equicontinuous set of $C(X)$ is contained in a τ_c-compact set by Ascoli's theorem, the fact that on the equicontinuous sets of $C(X)$ the pointwise and the compact-open topology coincide yields that every pointwise bounded equicontinuous set of $C(X)$ is τ_p-metrizable. So Theorem 2 applies to show that X satisfies the DCCC. $\square$

3. Some examples

According to [21, Theorem 3.5] a completely regular space X satisfies the DCCC if $C_p(X)$ is angelic, although the converse is not true. Since $C_p(X)$ is angelic if X is web-compact [18, Theorem 3], every web-compact space satisfies the DCCC. On the other hand, by [1, Proposition I.4.4] a completely regular space X also satisfies the DCCC if $C_p(X)$ is Lindelöf (the converse fails for instance for the 'double arrow' compact X).

Example 5. *If X satisfies the DCCC, each compact set of $C_c(X)$ need not be metrizable.* Let $I := D(\aleph_1)$ be the discrete space of first uncountable cardinality and set $X = L_p(I)$. According to [13, Example 6.2] there exists a compact set K in $C_c(X)$ that is not countably tight, therefore non-metrizable. But, according to [1, Corollary 0.5.19], the Suslin number of X is countable, i.e. $c(X) = \aleph_0$, so X satisfies the DCCC. Let us see that X is not a k-space. First note that X can be represented as the direct sum E of $\aleph_1$ copies of $\mathbb{R}$ equipped with the weak topology $\sigma(E, F)$, where $F = \mathbb{R}^{\omega_1}$ with ω_1 the first ordinal of cardinality $\aleph_1$, so that $X = (E, \sigma(E, F))$. If we equip E with the strongest locally convex topology $\mu(E, F)$, both topologies $\sigma(E, F)$ and $\mu(E, F)$ have the same bounded sets, which means that every compact set of $(E, \sigma(E, F))$ is finite-dimensional [16, 18.5(6)]. Since $\mu(E, F)$ is the inductive limit topology of all the finite-dimensional linear subspaces of E under their (unique) Euclidean topology [19, Chapter 5], it follows that $\sigma(E, F)$ and

$\mu(E,F)$ have the same compact sets. Given that $\mu(E,F) \neq \sigma(E,F)$, the k-extension of $\sigma(E,F)$, i.e. of the original topology of X, is strictly stronger than $\sigma(E,F)$. Therefore X cannot be a k-space.

Example 6. *If X is a k-space, each compact set of $C_c(X)$ need not be metrizable.* Let $X = D(\aleph_1)$. Then X satisfies the first axiom of countability and hence is a k-space, but since $C_c(X) = C_p(X) = \mathbb{R}^{\omega_1}$, the cube $[0,1]^{\omega_1}$ is a non-metrizable compact set of $C_c(X)$. Observe that if d is the trivial metric on X, the family $\{N_\epsilon : \epsilon > 0\}$ where $(x,y) \in N_\epsilon$ if $d(x,y) < \epsilon$, is a base of an admissible uniformity on X which is not trans-separable, otherwise X would be countable. Nonetheless the uniform structure $\mathcal{C}$ on X described in [11, 15.5] generates an admissible trans-separable uniformity on X. Actually, this is not the only admissible trans-separable uniformity on X. Another one is exhibited in [10, Example 6]. Since X is uncountable and discrete, X does not satisfy the DCCC.

Example 7. *If every compact set of $C_c(X)$ is metrizable, X need not be a k-space.* Let $\mathbb{N}$ be equipped with the discrete topology and choose $p \in \beta\mathbb{N} \setminus \mathbb{N}$. Then $X := \mathbb{N} \cup \{p\}$ with the relative topology of $\beta\mathbb{N}$ is not discrete and, using the fact that each open neighborhood of p meets $\mathbb{N}$ in a set belonging to the corresponding free ultrafilter $\mathcal{A}_p$ on $\mathbb{N}$ such that $\mathcal{A}_p \to p$ in $\beta\mathbb{N}$, it can be easily shown that every compact set of X is finite. This prevents X to be a k-space, because the finest topology on X with the same compact sets is the discrete topology. Since $C_p(X)$ is a subspace of $\mathbb{R}^X$, then $C_p(X)$ is metrizable. Therefore every compact set of $C_c(X) = C_p(X)$ is metrizable.

Example 8. *If X is a k-space and all compact sets of $C_c(X)$ are metrizable, X need not be realcompact.* According to [2, Example 7.14] there exists a Talagrand compact space K with a point $x \in K$ such that K coincides with the Stone–Čech compactification of $X = K \setminus \{y\}$. It turns out that $C_p(X)$ is K-analytic, hence angelic [8, Claim 4] and Lindelöf. Thus X satisfies the DCCC and every compact set in $C_c(X)$ is metrizable. Since X is pseudocompact but not compact, it is not realcompact. Observe that X, as a subspace of a Talagrand compact space, is Fréchet–Urysohn, hence a k-space.

Example 9. *If X is a k-space and all compact sets of $C_c(X)$ are metrizable, X need not be countably tight.* Split $\mathbb{N}$ provided with the discrete topology into two disjoint infinite sets A and B, and put $T = \overline{A}^{\beta\mathbb{N}}$. Then choose $y \in \overline{B}^{\beta\mathbb{N}} \setminus \mathbb{N}$ and define $X = \beta\mathbb{N} \setminus \{y\}$. Clearly X is Čech-complete, so a k-space. Moreover X is not realcompact. Clearly T is homeomorphic to $\beta\mathbb{N}$ and $y \notin T$ since $\overline{A}^{\beta\mathbb{N}} \cap \overline{B}^{\beta\mathbb{N}} = \emptyset$. Hence X contains a copy of $\beta\mathbb{N}$ and is not countably tight. Asanov's theorem [1, Theorem I.4.1] prevents $C_p(X)$ to be Lindelöf. Since X is separable, each compact set of $C_c(X)$ is metrizable.

Example 10. *If X is a Lindelöf P-space, each compact set of $C_c(C_p(X))$ is metrizable.* If X is a Lindelöf P-space, then $C_p(X)$ is Fréchet–Urysohn [1, Theorem II.7.15], hence a k-space. Since $c(C_p(X)) \leq \aleph_0$, [1, Corollary 0.3.7], it turns out that $C_p(X)$ satisfies the DCCC. Consequently, by Theorem 4, every compact set of $C_c(C_p(X))$ is metrizable.

Example 11. *If the compact sets of $C_c(C_p(X))$ are metrizable, the compact sets of $C_p(C_p(X))$ need not be metrizable.* Let X be a scattered compact space which is not metrizable. Then $C_p(X)$ is a k-space by [1, Theorem III.1.2], and the argument of the previous example shows that every admissible uniformity on $C_p(X)$ is trans-separable. This implies that every compact set of $C_c(C_p(X))$ is metrizable. But X is homeomorphic to a non-metrizable compact subset of $C_p(C_p(X))$.

Problem 12. Assume that X is completely regular but not a k-space. If every admissible uniformity on X is trans-separable, does satisfy X the DCCC?

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