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Metrization conditions for topological vector spaces with Baire type properties [☆]

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We show: (i) A Baire topological vector space is metrizable if and only if it has countable cs^*-character. (ii) A locally convex b-Baire-like space is metrizable if and only if it has countable cs^*-character. Both results extend earlier metrization theorems involving the concept of the cs^*-countable character. Theorem (ii) extends a theorem (Sakai) stating that the space $C_p(X)$ has countable cs^*-character if and only if X is countable.

1. Introduction

All topological spaces and groups are assumed to be Hausdorff. Good sufficient conditions for topological spaces and groups to be metrizable provide a significant problem. The classical metrization theorem of Birkhoff and Kakutani states that a topological group G is metrizable if and only if G is first-countable, i.e. G has a countable open base at the unit e of G.

Banakh and Zdomskyy [1] provided another sufficient condition for a Fréchet-Urysohn topological group to be metrizable. Let x be a point of a topological space X. A countable family $\mathcal{D}_x$ of subsets of X containing x is called a *countable cs^*-network at x* if for each sequence in X converging to x and each neighborhood

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U of x, there is $D \in \mathcal{D}_x$ such that $D \subseteq U$ and D contains infinitely many elements of that sequence. Following [1], a space X has *countable cs^*-character* if it has a countable cs^*-network at each point $x \in X$. Recall two results of Banakh and Zdomskyy.

Theorem 1.1. *([1]) (i) A topological group G is metrizable if and only if G is Fréchet–Urysohn and has countable cs^*-character. (ii) A Baire topological group G is metrizable if and only if G is sequential and has countable cs^*-character.*

Sakai [7] showed that the Fréchet–Urysohness in Theorem 1.1 can be replaced by an essentially weaker condition. Following Arhangel'skii, a topological space X is said to be *κ-Fréchet–Urysohn* if for every open subset U of X and every $x \in \overline{U}$, there exists a sequence $\{x_n\}_{n\in\mathbb{N}} \subseteq U$ converging to x. Note that the class of κ-Fréchet–Urysohn spaces is much wider than the class of Fréchet–Urysohn spaces [4]. Next follows immediately from [7, Lemma 2.14].

Theorem 1.2. *([7]) A topological group G is metrizable if and only if it is κ-Fréchet–Urysohn with countable cs^*-character.*

It turns out that, if the space $C_p(X)$ has countable cs^*-character, then $C_p(X)$ is already a κ-Fréchet–Urysohn space by [7, Lemma 2.11]. Making use of this fact and previous Theorem 1.2, Sakai proved the next remarkable result.

Theorem 1.3. *([7]) The space $C_p(X)$ is metrizable if and only if it has countable cs^*-character.*

Let us mention that the picture for the space $C_c(X)$ of all continuous real-valued functions on X with the compact-open topology is completely different as shown in [9] (see also [2]): for any Polish non-locally compact space X, the space $C_c(X)$ has countable cs^*-character and it is not metrizable.

These three theorems provide a nice application as well as the importance of the concept of countable cs^*-character. Our Theorem 1.4 characterizes countable cs^*-character at any point of a topological space.

We need a few technical notations. Let the product $\mathbb{N}^\mathbb{N}$ be endowed with the natural partial order, i.e., $\alpha \le \beta$ if $\alpha_i \le \beta_i$ for all $i \in \mathbb{N}$, where $\alpha = (\alpha_i)_{i\in\mathbb{N}}$ and $\beta = (\beta_i)_{i\in\mathbb{N}}$. For every $\alpha = (\alpha_i)_{i\in\mathbb{N}} \in \mathbb{N}^\mathbb{N}$ and each $k \in \mathbb{N}$, set $I_k(\alpha) := \{\beta \in \mathbb{N}^\mathbb{N} : \beta_i = \alpha_i \text{ for } i = 1, \ldots, k\}$.

Theorem 1.4. *Let x be a point of a topological space X. Then X has countable cs^*-character at x if and only if there is a family $\{A_\alpha : \alpha \in \mathbf{M}_x\}$ of subsets of X containing x such that*

- (i) *$\mathbf{M}_x$ is a subset of the partially ordered set $\mathbb{N}^\mathbb{N}$.*
- (ii) *$A_\beta \subseteq A_\alpha$, whenever $\alpha \le \beta$ for $\alpha, \beta \in \mathbf{M}_x$.*
- (iii) *For any neighborhood W at x there is $\alpha \in \mathbf{M}_x$ such that $A_\alpha \subseteq W$.*
- (iv) *For each $\alpha \in \mathbf{M}_x$ and each sequence S converging to x, there is $k \in \mathbb{N}$ such that $D_k(\alpha) \cap S$ is infinite, where $D_k(\alpha) := \bigcap_{\beta\in I_k(\alpha)\cap\mathbf{M}_x} A_\beta$.*

In this case the countable family $\mathcal{D} := \{D_k(\alpha) : \alpha \in \mathbf{M}_x \text{ and } k \in \mathbb{N}\}$ is a countable cs^*-network at x.

Arguably the most important class of topological groups is the class of topological vector spaces (tvs for short). Thus metrization theorem which supplements already known results might be of great use. Sakai [6, Proposition 2.5] proved that if $C_c(X)$ is Baire, then it is κ-Fréchet–Urysohn. This combined with Theorem 1.1(i) implies that every Baire space $C_c(X)$ with countable cs^*-character is metrizable. Our next result extends this theorem to any Baire tvs. Also Theorem 1.4 allows to extend Theorem 1.1(ii) in the realm of tvs by removing the assumption G is sequential.

Theorem 1.5. *A Baire tvs E is metrizable if and only if E has countable cs^*-character.*

It is also known (see [3, 2.8]) that $C_p(X)$ enjoys another nice (linear) topological property (which will be used in the sequel): For every increasing sequence $\{A_n\}_{n\in\mathbb{N}}$ of absolutely convex and closed sets covering $C_p(X)$ which is bornivorous (i.e., every bounded set in $C_p(X)$ is contained in some A_m) contains a member which is a neighborhood of zero. Locally convex spaces E (lcs for short) with this property are called *b-Baire-like*. Recall also (see [8]) that a lcs E is called *Baire-like* if every increasing sequence $\{A_n\}_{n\in\mathbb{N}}$ of absolutely convex closed subsets covering E contains a member which is a neighborhood of zero. Clearly, Baire lcs $\Rightarrow$ Baire-like $\Rightarrow$ b-Baire-like, the converse fails. Being inspired by Theorem 1.3 we prove the following

Theorem 1.6. *A b-Baire-like lcs E is metrizable if and only if it has countable cs^*-character. Consequently, $C_p(X)$ is metrizable if and only if it has countable cs^*-character.*

We know that every Fréchet–Urysohn lcs is b-Baire-like [3]. The next proposition improves this fact.

Proposition 1.7. *Each κ-Fréchet–Urysohn lcs is b-Baire-like.*

We note that there are b-Baire-like lcs which are not κ-Fréchet–Urysohn. For instance, Sakai [5] proved that $C_p(\mathbb{R})$ is *not* κ-Fréchet–Urysohn, however $C_p(\mathbb{R})$ is b-Baire-like by [3, 2.8]. This shows that Theorem 1.6 essentially extends Theorem 1.2 in the realm of locally convex spaces.

Our approach for Theorem 1.5 and Theorem 1.6 is natural, heavily depends on Theorem 1.4, and essentially differs from the proofs of Banakh–Zdomskyy's Theorem 1.1 and Sakai's Theorem 1.3.

2. Proof of Theorem 1.4

We are at position to start with the proof of Theorem 1.4.

Proof. *Necessity*. Assume that X has countable cs^*-character at x with a cs^*-network $\{D_i\}_{i\in\mathbb{N}}$, where D_i contains x for every $i \in \mathbb{N}$. If x is an isolated point, we set $\mathbf{M}_x := \mathbb{N}^\mathbb{N}$ and $A_\alpha := \{x\}$ for each $\alpha \in \mathbf{M}_x$. Clearly, the family $\{A_\alpha : \alpha \in \mathbf{M}_x\}$ satisfies (i)–(iv). Now we assume that x is not isolated. We prove the necessity in 3 steps.

Step 1. (See [2].) For every $k, i \in \mathbb{N}$, set

$$D_k^i := \bigcap_{l=1}^{k} D_{i-1+l}.$$

So, for each $i \in \mathbb{N}$, the sequence $\{D_k^i\}_{k\in\mathbb{N}}$ is decreasing. For every $\alpha = (\alpha_i)_{i\in\mathbb{N}} \in \mathbb{N}^\mathbb{N}$, set

$$A_\alpha := \bigcup_{i\in\mathbb{N}} D_{\alpha_i}^i = \bigcup_{i\in\mathbb{N}} \bigcap_{l=1}^{\alpha_i} D_{i-1+l}.$$

Clearly, $x \in A_\alpha$ and $A_\alpha \subseteq A_\beta$ for each $\alpha, \beta \in \mathbb{N}^\mathbb{N}$ with $\beta \leq \alpha$.

Step 2. Let V be a neighborhood of x. Set $J(V) := \{j \in \mathbb{N} : D_j \subseteq V\}$. Since x is not isolated, the family $J(V)$ is infinite. We prove also a few more conditions.

(**A**) If W is a neighborhood of x and $J(W) := \{j \in \mathbb{N} : D_j \subseteq W\} = \{n_k\}_{k\in\mathbb{N}}$ with $n_1 < n_2 < \ldots$, then there is $\alpha = \alpha(W) \in \mathbb{N}^\mathbb{N}$ such that

(A_1) $\alpha_{n_k} = 1$ for every $k \in \mathbb{N}$;
(A_2) $A_\alpha = \bigcup_{k\in\mathbb{N}} D_{n_k} (\subseteq W)$;
(A_3) $X \setminus A_\alpha$ does not contain sequences converging to x.

We construct $\alpha = \alpha(W)$ as follows. If $i = n_k$ for some $k \in \mathbb{N}$ we set $\alpha_i = 1$. So $D^i_{\alpha_i} = D_{n_k}$. Set $n_0 := 0$. Now, if $n_{k-1} < i < n_k$ for some $k \in \mathbb{N}$, we set $\alpha_i := n_k - i + 1$. Then

$$D^i_{\alpha_i} = \bigcap_{l=1}^{\alpha_i} D_{i-1+l} \subseteq D_{i-1+\alpha_i} = D_{n_k}.$$

Hence $A_\alpha = \bigcup_{k\in\mathbb{N}} D_{n_k}$. Thus ($A_1$) and ($A_2$) are satisfied.

Now we check (A_3). Suppose for a contradiction that there is a sequence $S \subseteq X \setminus A_\alpha$ which converges to x. By the definition of *cs**-network, there is $i \in \mathbb{N}$ such that $D_i \subseteq W$ and $D_i \cap S$ is infinite. In particular, $i \in J(W)$ and hence $D_i \subseteq A_\alpha$. So $A_\alpha \cap S$ is infinite, a contradiction.

Step 3. Denote by $\mathbf{M}_x$ the set of all $\alpha \in \mathbb{N}^{\mathbb{N}}$ of the form $\alpha = \alpha(W)$ for some neighborhood W of x. Clearly, (i) and (ii) hold. Condition (**A**) implies that the family $\{A_\alpha : \alpha \in \mathbf{M}_x\}$ satisfies also (iii). Let us check (iv).

Let S be a sequence converging to x. If S is trivial, (iv) follows from the fact that $D_k(\alpha)$ contains x for each $k \in \mathbb{N}$. Assume that S is not trivial. Let $\alpha = \alpha(W)$ for some neighborhood W of x. By the definition of *cs**-network there exists $i \in \mathbb{N}$ such that $D_i \subseteq W$ and $D_i \cap S$ is infinite. By the definition of $J(W)$ we have $i = n_k \in J(W)$ for some $k \in \mathbb{N}$. So $D_i = D_{n_k}$. As

$$\begin{aligned} D_{n_k}(\alpha) &= \bigcap_{\beta\in I_{n_k}(\alpha)\cap\mathbf{M}_x} A_\beta = \bigcap_{\beta\in I_{n_k}(\alpha)\cap\mathbf{M}_x} \left(\bigcup_{i\in\mathbb{N}} \bigcap_{l=1}^{\beta_i} D_{i-1+l}\right) \quad (\text{take } i = n_k) \\ &\supseteq \bigcap_{\beta\in I_{n_k}(\alpha)\cap\mathbf{M}_x} \left(\bigcap_{l=1}^{\beta_{n_k}} D_{n_k-1+l}\right) \quad \big(\text{since } \beta_{n_k} = \alpha_{n_k} = 1 \text{ by } (A_1)\big) = D_{n_k}, \end{aligned}$$

we obtain that $S \cap D_{n_k}(\alpha)$ is infinite. Setting $m = n_k$ we obtain the desired claim. So the necessity is proved.

Note that the following condition holds.

(**D**) $A_\alpha = \bigcup_{k\in\mathbb{N}} D_k(\alpha)$ for every $\alpha \in \mathbf{M}_x$.

A similar condition has been essentially used in [2].

Clearly, $\bigcup_{k\in\mathbb{N}} D_k(\alpha) \subseteq A_\alpha$. We prove the converse inclusion as follows

$$\begin{aligned} \bigcup_{k\in\mathbb{N}} D_k(\alpha) &= \bigcup_{k\in\mathbb{N}} \bigcap_{\beta\in I_k(\alpha)\cap\mathbf{M}_x} A_\beta \\ &= \bigcup_{k\in\mathbb{N}} \bigcap_{\beta\in I_k(\alpha)\cap\mathbf{M}_x} \left(\bigcup_{i\in\mathbb{N}} \bigcap_{l=1}^{\beta_i} D_{i-1+l}\right) \quad (\text{take only } i = k) \\ &\supseteq \bigcup_{k\in\mathbb{N}} \bigcap_{\beta\in I_k(\alpha)\cap\mathbf{M}_x} \left(\bigcap_{l=1}^{\beta_k} D_{k-1+l}\right) \quad (\text{since } \beta_k = \alpha_k) \\ &= \bigcup_{k\in\mathbb{N}} \bigcap_{l=1}^{\alpha_k} D_{k-1+l} = A_\alpha. \end{aligned}$$

Sufficiency. We show that the family $\mathcal{D}$ is a countable cs^*-network at x.

Let W be a neighborhood at x and S be a sequence converging to x. By (iii) we can choose $\alpha \in \mathbf{M}_x$ such that $A_\alpha \subseteq W$. Now (iv) implies that $S \cap D_k(\alpha)$ is infinite for some natural number k. □

In what follows we shall use the following lemma.

Lemma 2.1. *([7, Lemma 2.13]) If G is a κ-Fréchet–Urysohn group, then it is strongly κ-Fréchet–Urysohn, i.e., for each decreasing open sequence $\{U_n\}_{n\in\mathbb{N}}$ of G with $e \in \bigcap_{n\in\mathbb{N}} \overline{U_n}$, there are $g_n \in U_n, n \in \mathbb{N}$, converging to e.*

Using Theorem 1.4 and Lemma 2.1 we provide another proof of Theorem 1.2.

Proof of Theorem 1.2. We prove only the sufficiency. By Lemma 2.1 we can assume that G is a strongly κ-Fréchet–Urysohn topological group with countable cs^*-character at the unit e.

Let $\mathbf{M}_e$ be the family defined in Theorem 1.4 at e. Clearly, the family

$$\{\overline{D_k(\alpha)} : \ \alpha \in \mathbf{M}_e, \ k \in \mathbb{N}\}$$

is countable. So, by (iii) of Theorem 1.4 and since G is a Tychonoff space, it is enough to show that for each $\alpha \in \mathbf{M}_e$ there is $m \in \mathbb{N}$ such that $\overline{D_m(\alpha)}$ is a neighborhood of e.

Suppose for a contradiction that for some $\alpha \in \mathbf{M}_e$ all the sets $\overline{D_k(\alpha)}$ are not neighborhoods of e. Set $U_k := G \setminus \overline{D_k(\alpha)}$. Then $e \in \overline{U_k}$ for every $k \in \mathbb{N}$. As the sequence $\{D_k(\alpha)\}_{k\in\mathbb{N}}$ is increasing, the sequence $\{U_k\}_k$ is decreasing. So there are $g_n \in U_n, n \in \mathbb{N}$, converging to e. By Theorem 1.4(iv), there is $m \in \mathbb{N}$ such that $D_m(\alpha)$ contains infinitely many elements g_n, a contradiction. □

3. Proofs of Theorems 1.5 and 1.6

Dealing with topological vector spaces E one can fix further properties of the family $\{A_\alpha : \alpha \in \mathbf{M}_0\}$ useful in the sequel, where $\mathbf{0}$ is the neutral element of E. For a subset B of a tvs E and a natural number k we set $kB := \{kx : x \in B\}$. We need the following

Lemma 3.1. *Let E be a tvs of countable cs^*-character with a countable cs^*-network $\{D_k(\alpha) : \alpha \in \mathbf{M}_0, k \in \mathbb{N}\}$ at $\mathbf{0}$ as defined in Theorem 1.4. For each $\alpha \in \mathbf{M}_0$, the countable family $\{kD_k(\alpha)\}_{k\in\mathbb{N}}$ absorbs all bounded sets of E. In particular, $E = \bigcup_{k\in\mathbb{N}} kD_k(\alpha)$.*

Proof. Assume that there exists a bounded subset B of E such that $B \not\subset kD_k(\alpha)$ for every $k \in \mathbb{N}$. Then there is $f_k \in B$ such that $f_k \notin kD_k(\alpha)$. Hence $k^{-1}f_k \notin D_k(\alpha)$ for every $k \in \mathbb{N}$. Then $S := \{k^{-1}f_k\}_{k\in\mathbb{N}}$ is a non-trivial sequence converging to $\mathbf{0}$. Since the sequence $\{D_k(\alpha)\}_k$ is increasing, $S \cap D_k(\alpha)$ is *finite for every* $k \in \mathbb{N}$. On the other hand, Theorem 1.4(iv) implies that there is $m \in \mathbb{N}$ such that $D_m(\alpha) \cap S$ is *infinite*, a contradiction. □

Now we are ready to prove Theorems 1.5 and 1.6.

Proof of Theorem 1.5. Assume that E has countable cs^*-character. By Theorem 1.4 the space E has a family $\{A_\alpha : \alpha \in \mathbf{M}_0\}$ of subsets enjoying conditions (i)–(iv) at $\mathbf{0}$. Fix $\alpha \in \mathbf{M}_0$. Lemma 3.1 implies

$$E = \bigcup_{k\in\mathbb{N}} kD_k(\alpha) = \bigcup_{k\in\mathbb{N}} k\overline{D_k(\alpha)}.$$

Since E is Baire, there exists $k_0(\alpha) \in \mathbb{N}$ such that $\overline{D_k(\alpha)}$ has a nonempty interior for every $k \geq k_0(\alpha)$. Now condition (iii) of Theorem 1.4 applies to say that the countable family

$$\left\{\overline{D_k(\alpha)} - \overline{D_k(\alpha)} : \alpha \in \mathbf{M}_0, \ k \geq k_0(\alpha)\right\}$$

forms a base at zero in E. Thus E is metrizable. If E is metrizable it has a countable cs^*-character. □

Proof of Theorem 1.6. Assume E is a b-Baire-like space with countable cs^*-character. By Theorem 1.4 the space E has a family $\{A_\alpha : \alpha \in \mathbf{M}_0\}$ of its subsets satisfying conditions (i)–(iv) at $\mathbf{0}$. For every $\alpha \in \mathbf{M}_0$ denote by B_α the closure of the absolutely convex hull of A_α. So, for each $\alpha \in \mathbf{M}_0$ and every $k \in \mathbb{N}$ the sets

$$B_\alpha \quad \text{and} \quad D_{k,B}(\alpha) := \bigcap_{\beta \in I_k(\alpha) \cap \mathbf{M}_0} B_\beta$$

are closed and absolutely convex. Clearly, $D_k(\alpha) \subseteq D_{k,B}(\alpha)$ for each $\alpha \in \mathbf{M}_0$ and every $k \in \mathbb{N}$. Lemma 3.1 implies that the increasing sequence $\{kD_{k,B}(\alpha)\}_{k\in\mathbb{N}}$ of closed and absolutely convex sets absorbs all bounded sets of E. Since E is b-Baire-like, there is $k_0(\alpha) \in \mathbb{N}$ such that $kD_{k,B}(\alpha)$ is a neighborhood at $\mathbf{0}$ for every $k \geq k_0(\alpha)$. So $D_{k,B}(\alpha) = k^{-1} \cdot kD_{k,B}(\alpha)$ is also a neighborhood at $\mathbf{0}$ for every $k \geq k_0(\alpha)$. Now condition (iii) of Theorem 1.4 implies that the countable family

$$\left\{D_{k,B}(\alpha) : \alpha \in \mathbf{M}_0 \text{ and } k \geq k_0(\alpha)\right\}$$

is a base at $\mathbf{0}$. Thus E is metrizable. Clearly if E is metrizable, then E has countable cs^*-character and it is b-Baire-like by [3, 2.11]. □

Corollary 3.2. *Let $E = \prod_{j\in J} E_j$, where E_j is a metrizable lcs for every $j \in J$. Then E has countable cs^*-character if and only if the set J is countable.*

Proof. Note that E is b-Baire-like. If E has countable cs^*-character, then E is metrizable by Theorem 1.6. So J is countable. The converse is trivial. □

Now we prove Proposition 1.7.

Proof of Proposition 1.7. Let E be a κ-Fréchet–Urysohn lcs. We have to show that E is b-Baire-like.

Suppose for a contradiction that E is not b-Baire-like. So there is an increasing sequence $\{A_n\}_{n\in\mathbb{N}}$ of absolutely convex and closed sets covering E which is bornivorous and such that A_n is not a neighborhood of zero for every $n \in \mathbb{N}$. Set $U_n := E \setminus A_n$. Then $\{U_n\}_{n\in\mathbb{N}}$ is a decreasing sequence of open sets such that $\mathbf{0} \in \bigcap_{n\in\mathbb{N}} \overline{U_n}$. By Lemma 2.1, there are $g_n \in U_n, n \in \mathbb{N}$, converging to $\mathbf{0}$. So $g_n \in U_k$ for every $n \geq k$, and $S := \{\mathbf{0}, g_n : n \in \mathbb{N}\}$ is a compact subset of E. As $\{A_n\}_n$ is bornivorous, there is $m \in \mathbb{N}$ such that $S \subseteq A_m$. Thus $S \cap U_m = \emptyset$, a contradiction. □

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