Chapter 1
Some Aspects in the Mathematical Work of Jerzy Kąkol

Manuel López-Pellicer and Santiago Moll

Abstract We present some selected topics of the research developed by Professor Jerzy Kąkol with his collaborators. After some comments on Kąkol's Ph.D. dissertation we group together the chosen contributions in three sections: Topological vector spaces, Descriptive Topology and Functional Analysis and Other aspects in Kąkol's work. For the sake of clarity the sections have been divided in subsections.

Keywords Analytic space · Baire type condition · Compactness · Descriptive topology · Fredholm operator · Functional analysis · Hahn–Banach extension property · Hewitt spaces · *K*-analytic · Locally convex space · Non-archimedean functional analysis · Resolution · Space of the continuous functions · Sequential condition · Topological vector space

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1.1 Kąkol's Ph.D. Dissertation

Professor Kąkol got his Ph.D. in 1980, 2 years after his graduation in the A. Mickiewicz University. In his fundamental work on locally m-convex algebras [80, 81], E. Michael gave sufficient conditions for the local m-convexity of an algebra

M. López-Pellicer
Department of Applied Mathematics and IUMPA, Universitat Politècnica de València, 46022 València, Spain
e-mail: mlopezpe@mat.upv.es

S. Moll (✉)
Department of Applied Mathematics, Universitat Politècnica de València, 46022 València, Spain
e-mail: sanmollp@mat.upv.es

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X endowed with the linear inductive limit topology associated with an increasing sequence $(X_n, \xi_n)_n$ of locally m-convex subalgebras of X, a line of research that was continued in the works [97, 98] of Warner. Kąkol's dissertation deals with a similar problem but in the frame of the so-called generalized inductive limits of topological algebras in the sense of Alexiewicz.

This approach provided more applications in the realm of equivalent inductive systems. The notion of a generalized inductive limit of locally convex spaces was introduced first by Garling and systematically studied by Adasch, Ernst, Keim, Roelcke, Ruess, Turpin and Wiweger with the theory of the so-called two-normed spaces in the sense of Alexiewicz [1].

The essential result of Kąkol Ph.D. dissertation was included in the following main result:

Theorem 1.1 (Kąkol) *If $\Gamma = \{(S_n, \xi_n) : n \in \mathbb{N}\}$ is an m-bounded inductive system on the algebra X, then (X, ξ_Γ) is a topological algebra, where ξ_Γ is the finest linear topology ξ_Γ on X such that $\xi_\Gamma | S_n \leq \xi_n$ for all $n \in \mathbb{N}$.*

This theorem provides several applications, yielding the following applicable fact which essentially extends some results of Michael.

Corollary 1.1 *If Γ is the usual inductive system of commutative Banach algebras on the algebra X with a unit, then (X, ξ_Γ) is a locally m-convex algebra.*

1.2 Topological Vector Spaces

1.2.1 Hahn–Banach Extension Property

A number of results published by Kąkol deal with non-locally convex spaces and Hahn–Banach type extension theorems. Recall that a topological vector space E is said to have the Hahn–Banach extension property if for every subspace F of E every continuous linear functional on F can be extended to a continuous linear functional on E. In this case the topological dual E' of E is separating, i.e. E' separates points of E from zero. Every locally convex space has the Hahn–Banach extension property. In [25], Duren, Romberg and Shields gave an example of a dual separating F-space E, i.e. a metrizable and complete topological vector space, such that E' is separating and E has a subspace M such that E/M has a closed weakly dense subspace, so that E does not have the Hahn–Banach extension property. Shapiro showed in [89] that every sequence space ℓ^p, $0 < p < 1$, also contains such subspaces. Finally, in 1974 Kalton, developing basic sequence techniques for F-spaces, proved that all F-spaces with the Hahn–Banach extension property are locally convex [70].

In 1992, Kąkol presented in [47] an elementary construction for an abundance of vector topologies ξ on a fixed infinite dimensional vector space E such that (E, ξ) does not have the Hahn–Banach extension property but the topological dual $(E, \xi)'$

separates points of E. This construction might be placed in any academic book devoted to the classical Hahn–Banach theorem.

Theorem 1.2 (Kąkol) *Let E be an infinite dimensional vector space and $\Delta := \{(x, x) : x \in E\}$ the diagonal of $E \times E$. For every non-zero linear functional f on Δ there always exist two vector topologies ξ_1 and ξ_2 on E such that f is $\xi_1 \times \xi_2$-continuous on Δ but f cannot be extended to a continuous linear functional on $(E, \xi_1) \times (E, \xi_2)$.*

1.2.2 The Mackey–Arens Theorem

The classical Mackey–Arens theorem states that for any locally convex space $E = (E, \xi)$ there exists always the *finest locally convex topology* $\mu(E, E')$ on E compatible with ξ, i.e. both ξ and $\mu(E, E')$ have the same continuous linear functionals. It is natural to ask whether for a topological vector space E there exists the finest vector topology compatible with the original one. In papers [44, 45] Kąkol deals with the problem of the existence on a topological vector space E of not locally convex vector topologies producing the same topological dual as the original topology of E. The author proved, among others, the following negative result concerning the Mackey–Arens theorem.

Theorem 1.3 (Kąkol) *Let ξ be a vector topology on a vector space X different from the finest one on X and compatible with a dual pair (X, Y), i.e. Y being the topological dual of (X, ξ). If (X, ξ) contains a dense subspace of infinite codimension, then X does not admit the finest vector topology compatible with (X, Y).*

Kąkol results on the Mackey–Arens theorem have been used by Chasco et al. in [14], Perez García and Schikhof in [85] and Khan in [72].

1.2.3 Basic Sequences and the Hahn–Banach Theorem

In the paper [56], Kąkol and Sorjonen being motivated by the fundamental 1974 Kalton theorem of [70] provided necessary and sufficient conditions for a metrizable topological vector space with a generalized form of the Hahn–Banach Extension Property to be locally convex. The main theorem lead to the following extension of Kalton's theorem.

Corollary 1.2 (Kąkol–Sorjonen) *A metrizable [and separable] topological vector space with the Markushevich–Hahn–Banach Extension Property [Hahn–Banach Extension property] is either locally convex or its completion is not dual-separating.*

It is interesting to mention here that, having in mind the above corollary, Kalton constructed in [71] a quasi-Banach space which contains no basic sequence. This led to provide a metrizable locally bounded (non-complete) and non-locally convex space with the Hahn–Banach Extension property.

1.2.4 Fredholm Operators

Professor Kąkol also published a few articles connected with the Theory of Operators. Let (X, τ) and (Y, ξ) be F-spaces, i.e. metrizable and complete topological vector spaces, and let $T : X \to Y$ be a continuous linear operator. Consider the following condition:

(i) *There exists a regular basic sequence $(x_n)_n$ in X such that $T(x_n) \to 0$.*

In [23] Drewnowski proved that a continuous linear operator T mapping a Banach space into another one is semi-Fredholm if and only if T fails to have property (i).

The following two results of Kąkol in [46] essentially extend known results, including Drewnowski's result, to semi-Fredholm operators acting between F-spaces, i.e. metrizable and complete topological vector spaces.

Theorem 1.4 (Kąkol) *Let X and Y be F-spaces and $T \in L(X, Y)$. Consider the following conditions:*

(a) *T is m-semi-Fredholm, i.e. the range of T is closed and its kernel is a minimal space.*
(b) *Every closed non-minimal subspace of X contains a closed non-minimal subspace G such that $T|G$ is an isomorphism.*
(c) *Does not exist a non-minimal closed subspace G of X such that $T|G$ is compact.*
(d) *Does not exist a strongly regular M-basic sequence $(x_n)_n$ in X such that $T(x_n) \to 0$.*

Then (a) $\Rightarrow$ (b) $\Rightarrow$ (c) $\Rightarrow$ (b). Moreover, if every minimal subspace of X is q-minimal, then (b) $\Rightarrow$ (a).

Additionally to (i) Kąkol consider the following conditions on the F-spaces (X, τ) and (Y, ξ) and on the continuous linear operator $T : X \to Y$:

(ii) *There exists on X a strictly weaker metrizable vector topology γ such that τ is γ-polar, i.e. τ has a base of γ-closed neighborhoods of zero, and $T : (X, \gamma) \to (Y, \xi)$ is continuous.*
(iii) *There exists a closed non-minimal subspace $G \subset X$ such that $T|G$ is compact.*
With these two conditions Kąkol obtain the following corollary with two interesting equivalences.

Corollary 1.3 (Kąkol) *Let X, Y be F-spaces and $T \in L(X, Y)$. Then conditions (i) and (ii) are equivalent. Moreover, if X or Y is locally bounded, then (i) and (iii) are equivalent.*

1.2.5 Baire Type Conditions and Baire-Like Spaces in Saxon' Sense

The classical *Baire category theorem* says that if X is either a complete metric space or a locally compact Hausdorff space, then the intersection of countable many dense, open subsets of X is a dense subset of X. This result provides many applications as the Closed Graph Theorem, the Open Mapping Theorem and the Uniform Boundedness Theorem. Spaces having the topological property stated in the conclusion of the Baire theorem are called Baire spaces.

A new line of research concerning Baire-type conditions started with Saxon in [87]. Motivated by earlier results of Amemiya and Kōmura in [2], Saxon introduced the class of *Baire-like spaces* as those locally convex spaces X for which every increasing sequence of closed absolutely convex subsets of X covering X contains one member with a non-empty interior (equivalently, that it is a neighborhood of zero). It turns out that any metrizable locally convex space is Baire-like if and only if it is barrelled. Saxon proved that even more is true: *A barrelled locally convex space X is Baire-like provided X does not contain a copy of φ,* i.e. the $\aleph_0$-dimensional vector space with the finest locally convex topology. Saxon also obtained the following deep Closed Graph Theorem for Baire-like spaces: *A linear map with closed graph from a Baire-like space into an* (LB)*-space is continuous*.

In [22] Dierolf and Kąkol, being motivated by works of De Wilde, introduced, characterized and studied the locally convex spaces E (under the name *s-barrelled spaces*) for which one has that *every linear map from E into a Fréchet space* (i.e. a locally convex F space) *with sequentially closed graph is continuous*.

They provided a general construction which led to concrete examples of linear maps into Fréchet spaces with non-closed but sequentially closed graphs, and this approach was applied to show that the De Wilde's closed graph theorem fails if X is the inductive limit of a family of Baire bornological locally convex spaces. This result was used to answer in the negative a 20 years old question posed by W. Roelcke.

A number of papers related to this subject were jointly published with Roelcke, where some results of Lurje in [73, 74] have been extended. For more details see the Kąkol and Roelcke papers [53–55].

Many historical facts about this line of research with a long list of contributors can be found in the report made by Kąkol in [59].

1.2.6 The Spaces $C_p(X, E)$ and $C_c(X, E)$

If X is a completely regular Hausdorff space and E is a locally convex space, then $C_p(X, E)$ and $C_c(X, E)$ are the spaces of all continuous E-valued maps endowed with the pointwise and compact-open topology, respectively.

A characterization of the barrelledness of $C_c(X, E)$ is due to Mendoza [79]. On the other hand, the space $C_p(X, E)$ *is barrelled if and only if* $C_p(X)$ *and* E *are barrelled*

[88, Theorem IV.7.2], while Buchwalter and Schmets proved in [10] that $C_p(X)$ *is barrelled if and only if every topologically bounded set in X is finite*.

A good sufficient condition for $C_p(X)$ or $C_c(X)$ to be a Baire space seems to be difficult to find. Earlier results by Gruenhage and Ma in [42], showed that if X is a locally compact space or a first-countable space (or more generally, a k-space) then $C_c(X)$ *is Baire if and only if* X *has the moving-off property*. Granado and Gruenhage in [39] show that this characterization holds for generalized ordered spaces. As a corollary they prove that if X is a locally compact generalized ordered space, $C_c(X)$ is a Baire space if and only if X is paracompact.

In [86] Pytkeev, slightly modifying the game $\Gamma(X)$ of Lutzer and McCoy in [75] obtained that $C_p(X)$ *is a first category space if and only if player* I *has a winning strategy in the game* $\Gamma_1(X)$ *and if and only if there exists a disjoint sequence of finite nonempty subsets* $(\Delta_n)_n$ *of* X*such that* $\sup_n \min\{|f(x)| : x \in \Delta_n\} < \infty$ *for any* $f \in C(X)$. In this paper it is also proved that *for a locally convex topological space* X *the space* $C_p(X)$ *is barrelled if and only if* X *is monotonically-convex Baire*. Let us recall that a locally convex space X is monotonically-convex Baire if it is not the union of an increasing sequence of closed convex nowhere dense subsets of X.

Kąkol has obtained nice results in $c_0(E)$, $\ell_\infty(E)$ and $C(X, E)$ related to Baireness and Baire-likeness. For instance in [61] it is proved the following theorem:

Theorem 1.5 (Kąkol–Gilsdorff–Sánchez-Ruiz)

(a) $c_0(E)$ *is Baire-like if and only if* E *is barrelled and the strong dual* E'_β *of* E *is strong fundamentally* ℓ_1*-bounded.*
(b) *If* $\ell_\infty(E)$ *is Baire-like, then* $c_0(E)$ *is Baire-like. If every bounded subset of* E *is precompact and* $c_0(E)$ *is Baire-like, then* $\ell_\infty(E)$ *is Baire-like.*

Recall that a topological space X is called Fréchet-Urysohn if for each subset $A \subset X$ and each $x \in \overline{A}$ there exists a sequence in A which converges to x. The following relation between Fréchet-Urysohn and Baire properties was obtained by Kąkol and Sánchez Ruiz in [58]:

Theorem 1.6 (Kąkol–Sánchez-Ruiz) *Every sequentially complete Fréchet-Urysohn topological vector space is Baire. Consequently, if* X *is a Lindelöf* P*-space and* E *is a Fréchet locally convex space, then every sequentially closed linear subspace of* $C_p(X, E)$ *is Baire and ultrabornological.*

This provides another simple proof of the classical Baire theorem: *Any product of metrizable and complete locally convex space is Baire*.

1.2.7 Strongly Hewitt Spaces

This line of research was continued by Kąkol and Śliwa in the paper [62]. In the class of realcompact or Hewitt spaces they introduced a new class of completely regular Hausdorff spaces, named *strongly Hewitt spaces*, as those spaces X such that for each sequence $D := (x_n)_n$ in $X^* := \beta X \setminus X$ there exists $f \in C(\beta X)$ which is positive on X and vanishes on a subsequence of D. If X is strongly Hewitt, then every infinite set $A \subset X^*$ contains an infinite subset S which is relatively compact in X^* and C^*-embedded in βX. In the following theorem they show that strongly Hewitt spaces are related also to Baire properties.

Theorem 1.7 (Kąkol–Śliwa)

(a) *A space X is strongly Hewitt if and only if it is Hewitt and X^* is countably compact. Hence every locally compact Hewitt space is strongly Hewitt.*
(b) *Every strongly Hewitt space of pointwise countable type is locally compact.*
(c) *Let X be strongly Hewitt and E a metrizable locally convex space. Then $C_c(X, E)$ is Baire-like if and only if E is barrelled. Hence, if X is locally compact, then $C_c(X, E)$ is Baire-like if and only if E is barrelled and X is Hewitt.*
(d) *There exists a locally compact strongly Hewitt space X such that $C_c(X)$ is not Baire.*

1.2.8 Jarchow Problem: (df)-spaces $C_c(X)$

It is well known that the strong dual E'_β of any bornological locally convex space E with a fundamental sequence of bounded sets is a Fréchet space. A locally convex space E is a *df-space* if it contains a fundamental sequence of bounded sets and every null sequence in E'_β is equicontinuous. The class of *df*-spaces contains Grothendieck's class of *DF*-spaces. In page 270 of Jarchow's book [43] was stated that "*nothing seems to be known of when precisely $C_c(X)$ is a df-space*", while in the proof of Theorem 12.4.1 it is shown that for an arbitrary locally convex space E the strong dual E'_β is Fréchet if and only if E equipped with its Mackey topology becomes a *df*-space. For $E = C_c(X)$, Mazón adds in [78] the equivalent condition that each regular Borel measure on X has compact support. In the paper [77] McCoy and Todd characterize those X for which the uniform dual of $C_c(X)$ is a Banach space. These three authors unearthed that the two conditions where equivalent to the Jarchow's ancient query.

In the remarkable paper [65] Kąkol, Saxon and Todd prove that the Jarchow problem was equivalent to the two mentioned conditions and also to the eight additional conditions given in the next main theorem.

Theorem 1.8 (Kąkol–Saxon–Todd) *The following statements about $E := C_c(X)$ are equivalent:*

(i) *E is a (df)-space.*
(ii) *E has a fundamental sequence of bounded sets and is ℓ^∞-barrelled.*
(iii) *$(E', \beta(E', E))$ is a Fréchet space.*
(iv) *$(E', \beta(E', E))$ is a Banach space and equals $(E', n(E', E))$.*

(v) $(E', n(E', E))$ *is a Banach space.*
(vi) $(E', \beta(E', E))$ *is docile and locally complete.*
(vii) $(E', \sigma(E', E))$ *is docile and locally complete.*
(viii) *For each sequence* $(\mu_n)_n$ *in* E' *there exists a sequence* $(t_n)_n \subset (0, 1]$ *such that* $\{t_n\mu_n\}_n$ *is equicontinuous.*
(ix) *X is pseudocompact and* $(E', \sigma(E', E))$ *is locally complete.*
(x) *Each regular Borel measure on X has compact support.*
(xi) *Every countable union of supports sets in X is relatively compact.*

They apply this equivalence to provide a space X for which $C_c(X)$ is a *df*-space but not a *DF*-space. This concrete space $C_c(X)$ provides an example of an ℓ^∞-barrelled space $C_c(X)$ which is not $\aleph_0$-quasibarrelled, answering an old Buchwalter–Schmets question posed in [10, Remark 1].

1.3 Descriptive Topology and Functional Analysis

1.3.1 Sequential Conditions for Topological Groups and Montel (DF) and (LM)-Spaces

A part of the research of Professor Kąkol deals with sequential conditions of topological spaces and topological groups, a field of topology intensively developed from many years. We analyze some of results from this area, which were jointly obtained by Kąkol and Saxon in their celebrated paper [63].

Following Webb [99] a topological space X is *sequential* if every sequentially closed subset is closed and X verifies property C_3 if the sequential closure of every subset is sequentially closed. Clearly if X is sequential and verifies C_3 then X is Fréchet-Urysohn. It is well known that the strong dual φ of the Fréchet space $\mathbb{K}^{\mathbb{N}}$ of all scalar sequences is an $\aleph_0$-dimensional, Montel, (*LB*) and (*DF*)-space. Nyikos showed in [83, Example 1] that the space φ is sequential but it is not Fréchet-Urysohn.

Yoshinaga proved in [100] that the strong dual of a Fréchet–Schwartz space is sequential and Webb, in [99], extended Yoshinaga result to strong duals of Fréchet–Montel spaces (equivalently, to Montel (*DF*)-spaces).

A natural question is whether the Yoshinaga–Webb result extends to the class of (*LB*)-spaces, (*DF*)-spaces or (*LM*)-spaces. The answers are negative. Kąkol and Saxon proved the converse of Webb's result stating in [63, Theorems 4.5 and 4.6] that:

Theorem 1.9 (Kąkol–Saxon)

(A) *For a dual metric space E the following assertions are equivalent:*

1. *E is sequential.*
2. *E is normable or it is a Montel (DF)-space.*

(B) *For an (LM)-space E the following assertions are equivalent:*

1. *E is sequential.*
2. *E is metrizable or is a Montel (DF)-space.*

From these results it follows an easy answer to Nyikos topological group question in [83, Problem 1]. In the same frame Kąkol and Saxon prove that *every sequential proper (LB)-space is Montel* and that *a quasibarrelled (DF)-space E has property C_3 if and only if it is metrizable* [63, Corollary 4.3]. This result improves the analogous Webb's property for bornological spaces.

The topological property C_3 has been also used by Kąkol to characterize metrizability for (*LM*)-spaces in [57].

A topological vector space *X* has property C_3^- if the sequential closure of every linear subspace is sequentially closed. This definition follows from a nice results of Bonet and Defant in [9] showing that *the only infinite-dimensional Silva space with property C_3^- is φ*. They also proved that *if E is an infinite dimensional nuclear (DF)-space different from φ, then E contains a subspace whose sequential closure is not sequentially closed*. Kąkol and Saxon in [63] also obtained that if *M* is any metrizable space, then the product $M \times \varphi$ has property C_3^- but not property C_3 and they got the following descriptive results for (*LB*) and (*LF*) spaces that have property C_3^-:

Theorem 1.10 (Kąkol–Saxon) *An (LB)-space $(E, \tau) = \lim(E_n, \tau_n)$ has property C_3^- if and only if E is isomorphic to some Banach step E_q, to φ, or to the product $E_q \times \varphi$.*

Theorem 1.11 (Kąkol–Saxon) *An (LF)-space $(E, \tau) = \lim(E_n, \tau_n)$ has property C_3^- if and only if E is isomorphic to some metrizable (LF)-space M, to φ, or to the product $M \times \varphi$.*

A topological space *X* has countable tightness if for each subset *A* and each point $a \in \overline{A}$ there exists a countable subset *B* in *A* such that $a \in \overline{B}$. Sequential spaces have countable tightness but the converse is not true. The following topological description of distinguished Fréchet spaces was obtained by Ferrando et al. in [26]:

Theorem 1.12 (Ferrando–Kąkol–López-Pellicer–Saxon) *A Fréchet space E is distinguished if and only if its strong dual E'_β has countable tightness.*

Based in this result, Ferrando et al. found in [27] that *a Fréchet space E is distinguished if and only if every bounded set in E'_β has countable tightness.*

1.3.2 K-Analytic, Analytic Spaces, Compact Coverings (Resolutions)

A number of papers of Professor Kąkol deal with the *K-analyticity* of a topological (vector) space *E* and the concept of a *resolution* on *E*, i.e. a family of sets

$\{K_\alpha : \alpha \in \mathbb{N}^\mathbb{N}\}$ such that $E = \cup_{\alpha\in\mathbb{N}^\mathbb{N}} K_\alpha$ and $K_\alpha \subset K_\beta$ if $\alpha \le \beta$. Compact resolutions, i.e. resolutions $\{K_\alpha : \alpha \in \mathbb{N}^\mathbb{N}\}$ whose members are compact sets, naturally appear in many situations in Topology and Functional Analysis. A resolution $\{K_\alpha : \alpha \in \mathbb{N}^\mathbb{N}\}$ in E swallows compact sets if for each compact subset K in E there exists K_α such that $K \subset K_\alpha$. It is an easy and elementary exercise to observe that any separable metric and complete space E admits a compact resolution swallowing compact sets. Very far from triviality is the remarkable theorem due to Christensen [15, 68, Theorems 3.3 and 6.1, respectively], stating that a separable metric topological space X is a Polish space if and only if X admits a compact resolution swallowing compact sets.

Talagrand proved in [90] that any K-analytic space admits a compact resolution. It is worthwhile to mention two results of Cascales and Orihuela stating that for many topological spaces X the existence of such a resolution is enough for X to be K-analytic [11, Corollary 1.1] or analytic [12, Theorem 15, extending Talagrand's result that every metric K-analytic space is analytic].

Quasi-(LB)-spaces are locally convex spaces that admit special resolutions consisting of Banach discs. Quasi-(LB)-spaces and its relationships with the Closed Graph Theorem were defined and studied by Valdivia in [95], where it is proved that a Baire quasi-(LB)-space is a Fréchet space.

Tkachuk in his paper [93] proved that if $C_p(X)$ is K-analytic and Baire, the space X is countable and discrete. Hence a K-analytic Baire space $C_p(X)$ is a separable Fréchet space. In fact Tkachuk's theorem follows from the following theorem, due to De Wilde and Sunyach in [21].

Theorem 1.13 (De Wilde–Sunyach) *A Baire K-analytic locally convex space is a separable Fréchet space.*

Lutzer, van Mill and Pol complete this picture in [76] exhibiting a countable space X, having a unique non-isolated point, such that $C_p(X)$ is a separable, metrizable, non-complete Baire space. By Theorem 1.13 X is not K-analytic.

This line of research was continued by Kąkol and López-Pellicer in the paper [66]. Their following result essentially extends De Wilde–Sunyach theorem.

Theorem 1.14 (Kąkol–López-Pellicer) *A Baire locally convex space F with a relatively countably compact resolution is a separable Fréchet space.*

The proof used the following technical result from the same paper.

Proposition 1.1 (Kakol–López-Pellicer) *Let E be a topological space which admits a weaker topology ξ generated by a metric d. Let F be a dense Baire subspace of E having a resolution consisting of closed sets in ξ. Then $E \setminus F$ is of first Baire category.*

The concept of the angelicity introduced by Fremlin in [36] provided a strong motivation for several lines of research concerning to some variants of compactness. In [84] Orihuela introduced a large class of topological spaces X (under the name *web-compact*) for which the space $C_p(X)$ is angelic. Orihuela's theorem covers many already known partial results of Eberlein–Šmulian type.

Kąkol, Ferrando, López-Pellicer and Śliwa studied the subclass of web-compact spaces formed by the spaces that admit a resolution of relatively countably compact sets, called *strongly web-compact spaces*, in their paper [32]. They found that the square of a strongly web-compact space need not be strongly web-compact. Their construction used some idea presented by Novák in [82] and they proved:

Example 1.1 (Ferrando–Kakol–Lopez-Pellicer–Sliwa) There exists a countably compact topological space G such that the product $G \times G$ cannot be covered by a resolution of relatively countably compact sets. Hence, there exists a quasi-Suslin space X such that $X \times X$ is not quasi-Suslin.

It turns out that *strongly web-compact spaces* can be used to extend some classical Closed Graph Theorems. In [94, Chap. 1, 4.2. (11)] Valdivia stated that a linear map with closed graph from a metrizable Baire locally convex space E into a quasi-Suslin locally convex space F is continuous. Drewnowski proved in [24] that every continuous linear map from a topological vector space having a compact resolution onto an F-space is open. The Theorem 1.15 from [30] extends Valdivia's and Drewnowski's results.

Theorem 1.15 (Ferrando–Kąkol–López-Pellicer) *Let E and F be topological vector spaces such that E is Baire and F admits a relatively countably compact resolution. If $f : E \to F$ is a linear map with closed graph, there is a sequence $(U_n)_n$ in the family $\mathfrak{F}(E)$ of zero neighborhoods of E such that for every $V \in \mathfrak{F}(F)$ there exists $m \in \mathbb{N}$ with $m^{-1}U_m \subset f^{-1}(V)$. If $E = F$, then E is a separable F-space.*

From this theorem they get the Corollary 1.4.

Corollary 1.4 (Ferrando–Kakol–López-Pellicer) *Every linear map f from a Baire topological vector space E into a topological vector space F whose graph G admits a relatively countably compact resolution is continuous. Hence, every linear map from a metrizable and complete topological vector space into a separable metrizable topological vector space whose graph admits a complete resolution is continuous.*

For spaces $C_p(X)$, Ferrando and Kąkol in [29] obtain the following equivalences:

Theorem 1.16 (Ferrando–Kąkol) *The following conditions are equivalent:*

1. *$C_p(X)$ admits a bounded resolution.*
2. *$C_p(X)$ is K-analytic-framed in $\mathbb{R}^X$, i.e. there exists a K-analytic space Z such that $C_p(X) \subset Z \subset \mathbb{R}^X$.*
3. *$C_p(X)$ is K-analytic-framed in $\mathbb{R}^X$ and $C_p(X)$ is angelic.*

This applies to provide simple, almost elementary, proofs of the following remarkable results due to Talagrand, Tkachuk and Velichko-Tkachuk-Shakhmatov [92], respectively:

(i) *Let X be a compact space. Then $C_p(X)$ is K-analytic if and only if $C_c(X)$ is weakly K-analytic* [90, Théorème 3.4].

(ii) *For any completely regular space* X, *the space* $C_p(X)$ *is* K*-analytic if and only if* $C_p(X)$ *has a compact resolution* [93, Theorem 2.8].
(iii) $C_p(X)$ *is covered by a sequence of compact (relatively countably compact) sets if and only if* X *is finite* [7, Theorem I.2.1 and Corollary I.2.4].

Ferrando gives the first contribution in [31] to the general problem of when analyticity or K-analyticity of the weak topology $\sigma(E, E')$ of a dual pair (E, E') can be lifted to a stronger topology on E compatible with the dual pair, by means of the following theorem.

Theorem 1.17 (Ferrando) *The dual of* $C_p([0,1])$ *endowed with the Mackey topology is a weakly analytic space that it is not* K*-analytic.*

This nice result was complemented in [67] with the following theorem:

Theorem 1.18 (Kąkol–López-Pellicer–Śliwa) *For a completely regular space* X, *the Mackey dual of* $C_p(X)$ *is analytic if and only if* X *is countable.*

1.3.3 Class $\mathfrak{G}$ and K-Analyticity

Let E be an (LM)-space, i.e. the inductive limit of a sequence of metrizable locally convex spaces $(E_k)_k$, with a countable base of absolutely convex neighborhoods of zero $(U_n^k)_n$ in each E_k such that $U_{n+1}^k \subset U_n^k$, for every $k, n \in \mathbb{N}$. For $\alpha = (n_k) \in \mathbb{N}^{\mathbb{N}}$ set $K_\alpha := \bigcap_k (U_{n_k}^k)^\circ$. The family $\{K_\alpha : \alpha \in \mathbb{N}^{\mathbb{N}}\}$ is a relatively countably compact resolution in $(E', \sigma(E', E))$ and each sequence in every K_α is equicontinuous.

Let E be a (DF)-space and let $(S_n)_n$ be a fundamental sequence of bounded sets. For $\alpha = (n_k) \in \mathbb{N}^{\mathbb{N}}$ set $K_\alpha := \bigcap_k n_k S_k^\circ$. Then $\{K_\alpha : \alpha \in \mathbb{N}^{\mathbb{N}}\}$ is as above.

Let F be a locally complete (LF)-space and let $E := (F', \beta(F', F))$ be its strong dual. Then F has a resolution $\{A_\alpha : \alpha \in \mathbb{N}^{\mathbb{N}}\}$ consisting of Banach discs and each bounded set in F is contained in some A_α. For $\alpha \in \mathbb{N}^{\mathbb{N}}$ set $K_\alpha := A_\alpha^{\circ\circ}$ in E' and $\left\{K_\alpha : \alpha \in \mathbb{N}^{\mathbb{N}}\right\}$ in $(E', \sigma(E', E))$ is as above. In particular this holds for the space of distributions $E := \mathfrak{D}'(\Omega)$ over an open set $\Omega \subset \mathbb{R}^n$, as well as for the space $A(\Omega)$ of real analytic functions on Ω.

The common topological structure that appears in the dual $(E', \sigma(E', E))$ of the above examples motivated Cascales and Orihuela in [12] to introduce the class $\mathfrak{G}$ of locally convex spaces, such that a locally convex space E belongs to class $\mathfrak{G}$ if $(E', \sigma(E', E))$ admits a resolution $\{A_\alpha : \alpha \in \mathbb{N}^{\mathbb{N}}\}$ with the property that each sequence in every A_α is equicontinuous. Therefore the locally convex spaces mentioned above belong to class $\mathfrak{G}$.

Class $\mathfrak{G}$ enjoys good permanence properties, being its precompact subsets metrizables [12, Theorem 2]. A general characterization for the metrization of precompacts subsets of a locally convex space was obtained in [28, Theorem 3]:

Theorem 1.19 (Ferrando–Kąkol–López-Pellicer) *Precompact sets in a locally convex space E are metrizable if and only if E' endowed with the topology τ_p of uniform convergence on the precompact sets of E is trans-separable.*

Nevertheless in [64, Corollary 2.8] Cascales, Kąkol and Saxon proved that $C_p(X) \in \mathfrak{G}$ if and only if X is countable. Theorems 2.2 and 2.5 of this paper established the following characterization of global nature of metrizability in the class $\mathfrak{G}$ that extends a number of metrization theorems. Recall that φ is the $\aleph_0$-dimensional vector space with the finest locally convex topology.

Theorem 1.20 (Cascales-Kąkol–Saxon) *For a locally convex space E in class $\mathfrak{G}$ the following conditions are equivalent:*

(i) *E is metrizable.*
(ii) *E is Fréchet-Urysohn.*

If, additionally E is barrelled, the preceding conditions are equivalent to the property that E does not contain φ.

The next two theorems are in [13, Theorems 4.6, 4.7 and 4.8]. Its motivation and proof comes in part from Cascales result that if $E \in \mathfrak{G}$ then $(E', \sigma(E', E))$ is quasi-Suslin [11, Proposition 1]. They are related to the inverse case of a following general problem due to Corson asking if it is true that the unit ball in E' has $\sigma(E', E)$ countable tightness when E is a weakly Lindelöf Banach space. This Corson problem seems to be still open.

Theorem 1.21 (Cascales-Kąkol–Saxon) *If $E \in \mathfrak{G}$ the following assertions are equivalent:*

1. $(E, \sigma(E, E'))$ *has countable tightness.*
2. $(E', \sigma(E', E))$ *is realcompact.*
3. $(E', \sigma(E', E))$ *is Lindelöf.*
4. $(E', \sigma(E', E))^n$ *is Lindelöf for each $n \in \mathbb{N}$.*
5. $(E', \sigma(E', E))$ *is K-analytic.*

Theorem 1.22 (Cascales-Kąkol–Saxon)

(1) *If $E \in \mathfrak{G}$ and has countable tightness then $(E, \sigma(E, E'))$ have countable tightness.*
(2) *If $E \in \mathfrak{G}$ and it is quasibarrelled then E has countable tightness.*

The next result obtained by Cascales and Orihuela in [12, Theorem 13] extends a deep theorem of Amir-Lindestrauss for weakly compact generated Banach spaces. Let us notice that locally convex Lindelöf Σ-spaces, called also weakly countably determined, have attracted specialists for many years because of some important applications in the theory of $C_p(X)$ spaces.

Theorem 1.23 (Cascales-Orihuela) *If the space E belongs to the class $\mathfrak{G}$ and $(E, \sigma(E, E'))$ is a Lindelöf Σ-space then* $\mathrm{dens}(E, \sigma(E, E')) = \mathrm{dens}(E', \sigma(E', E))$.

The next characterization for duals Lindelöf Σ-spaces was obtained by Ferrando and Kąkol in [33].

Theorem 1.24 (Ferrando–Kąkol) *Let E be a locally convex space such that every weak*-bounded set in E' is relatively weak*-compact. Then $(E, \sigma(E, E'))$ has a Σ-covering with limited envelope if and only if $\left(E', \sigma\left(E', E\right)\right)$ is a Lindelöf Σ-space.*

A particular case of a version of an Amir-Lindestrauss theorem-type for spaces $C_p(X)$ is given by Ferrando et al. in [35, Theorem 1]. With the use of Nagami index they obtain new topological cardinal inequalities for spaces $C_p(\upsilon_\lambda X)$. For instance it is stated that:

Theorem 1.25 (Ferrando–Kąkol–López-Pellicer–Muñoz) *If $L \subset C_p(X)$ is a Lindelöf Σ-space and the Nagami index $Nag(X)$ of X is less or equal than the density* $\mathrm{dens}(L)$ *of L (this holds for example if X is a Lindelöf Σ-space), then:*

(i) *There exists a completely regular Hausdorff space Y such that $Nag(Y) \le Nag(X)$, $L \subset C_p(Y)$ and* $\mathrm{dens}(Y) = \mathrm{dens}(L)$.
(ii) *Y admits a weaker completely regular Hausdorff topology τ' such that $w(Y, \tau') \le d(Y)$.*

A locally convex space E has a $\mathfrak{G}$-base if it admits a base of neighborhoods of zero $\{U_\alpha : \alpha \in \mathbb{N}^\mathbb{N}\}$ satisfying the decreasing condition $U_\beta \subset U_\alpha$ whenever $\alpha \leqslant \beta$ in $\mathbb{N}^\mathbb{N}$ [68, Lemma 15.2 (iii)]. If E has a $\mathfrak{G}$ -base then $E \in \mathfrak{G}$ but, in general, the converse is not true. Concerning to $\mathfrak{G}$-bases is worthwhile to mention the following result of Ferrando and Kąkol in [34].

Theorem 1.26 (Ferrando–Kąkol) *For a completely regular topological space X, the space $C_c(X)$ has a $\mathfrak{G}$-base if and only if X contains a compact resolution swallowing the compact subsets.*

Generalizing the notion of metrizability, Tsaban and Zdomskyy introduced in [91] the strong Pytkeev property in relation with some deep metrization problems. They proved that the space $C_c(X)$ has the strong Pytkeev property for any Polish space X. This theorem has been recently extended by Gabriyelyan et al. in their preprint [38] where it is proved:

Theorem 1.27 (Gabriyelyan–Kąkol–Leiderman) *Let X be a space admitting a compact resolution swallowing the compact sets. The following are equivalent:*

(i) *$C_c(X)$ has the strong Pytkeev property.*
(ii) *$C_c(X)$ has countable tightness.*
(iii) *$C_p(X)$ has countable tightness.*
(iv) *X is a μ-space (this condition is equivalent to the barrelledness of $C_c(X)$).*

In the same paper [38] Gabriyelyan, Kąkol and Leiderman found the following interesting relations between $\mathfrak{G}$-bases and strong Pytkeev property in topological groups.

Theorem 1.28 (Gabriyelyan–Kąkol–Leiderman) *A sequential topological group with a $\mathfrak{G}$-base has the strong Pytkeev property. Consequently, a Baire topological group G is metrizable if and only if G has a $\mathfrak{G}$-base and is sequential. Conversely, any topological group which has the strong Pytkeev property admits a base of neighborhoods of the unit of the form $\{U_\alpha : \alpha \in \mathbb{M}\}$, where $\mathbb{M}$ is a subset of the partially ordered set $\mathbb{N}^\mathbb{N}$.*

1.3.4 ℓ_c-Equivalence and Metrizability

Two spaces X and Y are said to be *ℓ_p-equivalent* if the spaces $C_p(X)$ and $C_p(Y)$ are linearly homeomorphic and, analogously, X and Y are *ℓ_c-equivalent* if the spaces $C_c(X)$ and $C_c(Y)$ are linearly homeomorphic.

Arhangel'skiĭ asked in [6] *if metrizability is preserved by ℓ_p-equivalence in the class of first countable spaces.* Baars et al. proved in [8, Theorem 3.3] that *complete metrizability is preserved by ℓ_p-equivalence in the class of metrizable spaces*, given an alternative proof in the class of separable metrizable spaces by using a classical result of Christensen [15, 68, Theorems 3.3 and 6.1, respectively]. Later on, Valov in [96, Corollary 4.6] proved that a Čech-complete and first countable space Y is metrizable when it is ℓ_p-equivalent to a metrizable space X.

In the class of first countable spaces the preservation by *ℓ_c-equivalence of separable metrizability as well as the ℓ_c*-invariance of Polish spaces have been considered very recently by Kąkol et al. in [69, Corollaries 22 and 23]. These two ℓ_c-invariant properties may be extended to the class of spaces of pointwise countable type, that contains also the class of locally compact spaces (see *op. cit.* Corollary 24 for the ℓ_c-invariance of Polish spaces).

1.4 Other Aspects in Kąkol Work

1.4.1 Quantitative Outlook of Compactness

In recent years, several quantitative counterparts of some classical theorems such as Krein-Šmulyan, Eberlein-Šmulyan and Grothendieck theorems have been proved by different authors. These new versions also are in the Kąkol's work, always focused on new problems and its applications in topology and analysis. In [3, 4] we may find among others the next results:

Theorem 1.29 (Angosto–Kąkol–López-Pellicer) *Let X be a web-compact space and (Z, d) a separable metric space, let $H \subset Z^X$ be a τ_p-relatively compact set and let $C(X, Z)$ be the space of Z-valued continuous functions defined on X. Then $ck(H) \leq \hat{d}(\overline{H}, C(X, Z)) \leq 17ck(H)$, where $ck(H)$ is the "worst" distance to $C(X, Z)$ of the* *set of cluster points in* Z^X *of sequences in* H. *Therefore the following conditions are equivalent:*

1. $ck(H) = 0$.
2. H *is a relatively countably compact subset of* $C(X, Z)$.
3. H *is a relatively compact subset of* $C(X, Z)$.

A quantitative sequential approximation of closure points [3, Theorem 15] enables us to give a nice proof of the well known Orihuela result about the angelicity of $C_p(X, Z)$ for web-compact X and Z metric [84, Corollary 17].

For a Fréchet space E we define $k(H) := \sup\{d(h, E) : h \in \overline{H}^{\sigma(E'', E')}\}$, where $d(h, E)$ is the natural distance from h to E in the bidual E''. As usual coH is the convex hull of H.

Theorem 1.30 (Angosto–Kąkol–Kubzdela–López-Pellicer) *For a bounded set* H *in a Fréchet space* E *the following inequality holds* $k(coH) < (2^{n+1} - 2)k(H) + \frac{1}{2^n}$ *for all* $n \in \mathbb{N}$. *Consequently, this yields also the following formula* $k(coH) \leq \sqrt{k(H)}(3 - 2\sqrt{k(H)})$.

Hence coH is weakly relatively compact provided H is weakly relatively compact in E. This theorem extends the quantitative version of Krein's theorem for Banach spaces obtained by Granero, Hájek and Montesinos Santalucía in [40, 41].

Two another measures of weak non-compactness $lk(H)$ and $k'(H)$ for a Fréchet space were discussed in [4] providing two quantitative versions of Krein's theorem for the both functions.

1.4.2 A Glance on Non-archimedean Functional Analysis in Kąkol Work

An essential part of the research of Professor Kąkol deals also with non-archimedean Functional Analysis, including some results about tensor products, Hahn–Banach type theorems and Dugundji Extension type theorems among others. Kąkol provided an essential application of so-called t-frames to the study of compactoids. His results may be found in [16–20, 60].

Since this line of research for non-specialists might provide some extra technical and theoretical difficulties, we refer to a recent monograph [85] by Perez Garcia and Schikhof, published in 2010, where many results of Kąkol have been presented. Kąkol results after 2010 may be found in his papers [5, 48–52].

It must be noted how far some non-archimedean cases might be from the classical ones. For instance Kąkol proved that a non-archimedean field $\mathbb{K}$ is spherically complete if and only if every locally convex space over $\mathbb{K}$ has the Hahn–Banach Extension Property.

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