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On Topological Properties of the Weak Topology of a Banach Space

Saak Gabrielyan

Department of Mathematics, Ben-Gurion University of the Negev, P. O. 653, Beer-Sheva, Israel
saak@math.bgu.ac.il

Jerzy Kakol

Faculty of Mathematics and Informatics, A. Mickiewicz University, Matejki 48-49, 60-769 Poznan, Poland
kakol@amu.edu.pl

Lyubomyr Zdomskyy

Kurt Gödel Research Center for Mathematical Logic, University of Vienna, Währinger Str. 25, 1090 Vienna, Austria
lyubomyr.zdomskyy@univie.ac.at

Being motivated by the famous Kaplansky theorem we study various sequential properties of a Banach space E and its closed unit ball B , both endowed with the weak topology of E . We show that B has the Pytkeev property if and only if E in the norm topology contains no isomorphic copy of l_1 , while E has the Pytkeev property if and only if it is finite-dimensional. We extend a result of G. Schlüchtermann and R. F. Wheeler [*The Mackey dual of a Banach space*, *Noti de Matematica XI* (1991) 273–287] by showing that B is a (separable) metrizable space if and only if it has countable cs^* -character and is a k -space. As a corollary we obtain that B is Polish if and only if it has countable cs^* -character and is Čech-complete, that supplements a result of G. A. Edgar and R. F. Wheeler [*Topological properties of Banach spaces*, *Pacific J. Math.* 115 (1984) 317–350].

Keywords: Weak topology, Banach space, aleph-space, k -space, cs^* -character.

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